PART - III

MATHEMATICS / رياضي (Urdu & English Version / اردو اور انگریزی ریاضي)

(Urdu & English Version / اردو اور انگریزی ریاضي)

Time Allowed : 2.30 Hours ]

[ Maximum Marks : 90

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - A / A-

20x1=20

(i) لوہت : (b) تھام پیچھے کی لون ہوئی ہے

(ii) لوہت : (b) تھام پیچھے کی لون ہوئی ہے

Note : (i) All questions are compulsory.

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.
The percentage error in the 11th root of the number 28 is approximately \( \frac{1}{11} \) times the percentage error in 28.

\[
\begin{align*}
&\text{(1) 11} &\quad \text{(2) 28} &\quad \text{(3) } \frac{1}{28} &\quad \text{(4) } \frac{1}{11}
\end{align*}
\]

The line \( 5x - 2y + 4k = 0 \) is a tangent to \( 4x^2 - y^2 = 36 \) if \( 5x - 2y + 4k = 0 \).

\[
\begin{align*}
&\text{(1) } \frac{9}{4} &\quad \text{(2) } \frac{81}{16} &\quad \text{(3) } \frac{4}{9} &\quad \text{(4) } \frac{2}{3}
\end{align*}
\]

In the multiplicative group of cube root of unity, the order of \( \omega^2 \) is:

\[
\begin{align*}
&\text{(1) 2} &\quad \text{(2) 1} &\quad \text{(3) 4} &\quad \text{(4) 3}
\end{align*}
\]

If \( f(x) \) and \( g(x) \) are two functions as defined in Generalized law of mean then Lagrange’s law of mean is a particular case of Generalised law of mean for:

\[
\begin{align*}
&\text{(1) } f’(x) = 0 &\quad \text{(2) } g’(x) = 0 &\quad \text{(3) } g(x) \text{ is an identity function} &\quad \text{(4) } f(x) \text{ is an identity function}
\end{align*}
\]
If $-x - iy$ lies in the first quadrant, then $-ix + y$ lies in the:

(1) third quadrant  (2) fourth quadrant  
(3) first quadrant  (4) second quadrant

Which of the following is a tautology?

(1) $p \lor (~p)$  (2) $p \land (~p)$  (3) $p \lor q$  (4) $p \land q$

Variance of the random variable $X$ is 4. Its mean is 2. Then $E(X^2)$ is:

(1) 6  (2) 8  (3) 2  (4) 4

$r = s \hat{i} - t \hat{k}$ is the equation of:

(1) $yz$-plane  (2) $xz$-plane  
(3) a straight line joining the points $\hat{i}$ and $\hat{k}$  (4) $xy$-plane
Which one of the following statements is true about the curve \( y = x^{\frac{1}{3}} \)?

1. The curve has a point of inflection in which \( y'' \) does not exist
2. The curve has more than one point of inflection
3. The curve has no point of inflection
4. The curve has a point of inflection in which \( y'' = 0 \)

If \( z_1 = 1 + 2i \), \( z_2 = 1 - 3i \) and \( z_3 = 2 + 4i \) then, the points on the Argand diagram representing \( z_1 z_2 z_3, 2z_1 z_2 z_3, -7z_1 z_2 z_3 \) are:

1. Vertices of an isosceles triangle
2. Collinear
3. Vertices of a right angled triangle
4. Vertices of an equilateral triangle
In the homogeneous system $\rho(A)$ is less than the number of unknowns, then the system has:

1. only non-trivial solutions
2. no solution
3. only trivial solution
4. trivial solution and infinitely many non-trivial solutions

$\frac{dy}{dx} = cx - c^2$ is the general solution of the differential equation:

1. $y' = c$
2. $(y')^2 + xy' + y = 0$
3. $(y')^2 - xy' + y = 0$
4. $y'' = 0$

The order and degree of the differential equation $y' + (y')^2 = x(x + y')^2$ are:

1. 1, 2
2. 1, 1
3. 2, 2
4. 2, 1
The value of \( \int_0^{\frac{\pi}{2}} \frac{\tan x - \cot x}{1 + \tan x \cot x} \, dx \) is:

\[(1) \frac{\pi}{4} \quad (2) \pi \quad (3) \frac{\pi}{2} \quad (4) 0 \]

In a Poisson distribution if \( P(X=2) = P(X=3) \) then, the value of its parameter \( \lambda \) is:

\[(1) 3 \quad (2) 0 \quad (3) 6 \quad (4) 2 \]

The surface area of the solid of revolution of the region bounded by \( x^2 + y^2 = 4, \ x = -2 \) and \( x = 2 \) about \( x \)-axis is:

\[(1) 64\pi \quad (2) 32\pi \quad (3) 8\pi \quad (4) 16\pi \]

If \( \vec{a} + \vec{b} + \vec{c} = 0, \ |\vec{a}| = 3, \ |\vec{b}| = 4, \ |\vec{c}| = 5 \) then, the angle between \( \vec{a} \) and \( \vec{b} \) is:

\[(1) \frac{5\pi}{3} \quad (2) \frac{\pi}{2} \quad (3) \frac{\pi}{6} \quad (4) \frac{2\pi}{3} \]
The tangents at the end of any focal chord to the parabola $y^2 = 12x$ intersect on the line:

(1) $y + 3 = 0$  (2) $y - 3 = 0$  (3) $x - 3 = 0$  (4) $x + 3 = 0$

If $A$ is a scalar matrix with scalar $k \neq 0$, of order 3, then $A^{-1}$ is:

(1) $\frac{1}{k^3} I$  (2) $kI$  (3) $\frac{1}{k^2} I$  (4) $\frac{1}{k} I$

The surface area of a sphere when the volume is increasing at the same rate as its radius, is:

(1) $4\pi$  (2) $\frac{4\pi}{3}$  (3) $1$  (4) $\frac{1}{2\pi}$
PART - II / II-

7x2 = 14

(i) Answer any seven questions.

(ii) Question number 30 is compulsory.

To find the number of coins, in each category, write the suitable system of equations for the given situation:

“A bag contains 3 types of coins namely ₹ 1, ₹ 2 and ₹ 5. There are 30 coins amounting to ₹ 100 in total.”

\[ m = \frac{2}{3} \]

If the two vectors \( \mathbf{3i} + \mathbf{2j} + \mathbf{9k} \) and \( \mathbf{i} + m \mathbf{j} + 3 \mathbf{k} \) are parallel, then prove that

\[ m = \frac{2}{3}. \]

\[ \left( \frac{1+i}{1-i} \right)^n = 1 \]

Find the least positive integer \( n \) such that \( \left( \frac{1+i}{1-i} \right)^n = 1 \).
Draw the diagram for the given situation:

“A comet is moving in a parabolic orbit around the sun which is at the focus of a parabola. When the comet is 80 million kms from the sun, the line segment from the sun to the comet makes an angle of \( \frac{\pi}{3} \) radians with the axis of the orbit.”

Find the critical numbers of \( f(x) = \sin x \).

Write the domain and extent of the function \( f(x) = x^3 + 1 \).

Prove that \( \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \csc x} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{1 + \tan x} \).

Show that the set of all non-zero rational numbers is not closed under addition.
Prove that \( F(3) = 1 - e^{-9} \) if the probability density function \( f(x) \) is defined as

\[
f(x) = \begin{cases} 
3e^{-3x}, & x > 0 \\
0, & x \leq 0
\end{cases}
\]

Verify Rolle’s theorem for the function \( f(x) = |x - 2| + |x - 5| \) in \([1, 6]\).

Note:
(i) Answer any seven questions.
(ii) Question number 40 is compulsory.

Prove that \( \rho(A) + \rho(B) \neq \rho(A + B) \) by giving the suitable matrices \( A \) and \( B \) of order 3.

Find the vectors of magnitude 6 which are perpendicular to both the vectors \( 4\mathbf{i} - \mathbf{j} + 3\mathbf{k} \) and \( -2\mathbf{i} + \mathbf{j} - 2\mathbf{k} \).
If \( n \) is a positive integer, prove that
\[
\left( \frac{1 + \sin \theta - i \cos \theta}{1 + \sin \theta + i \cos \theta} \right)^n = \cos n \left( \frac{\pi}{2} - \theta \right) - i \sin n \left( \frac{\pi}{2} - \theta \right)
\]

Show that the tangent to a rectangular hyperbola terminated by its asymptotes is bisected at the point of contact.

\[
\left( 0, \frac{\pi}{4} \right)
\]

Show that the function \( f(x) = \tan^{-1}(\sin x + \cos x), \ x > 0 \) is strictly increasing in the interval \( \left( 0, \frac{\pi}{4} \right) \).

If \( f(x, y) = \frac{1}{\sqrt{x^2 + y^2}} \) then, prove that \( x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = -f \).

Derive the formula for the volume of a cylinder with radius \( r \) and height \( h \) by using integration.

\[
(p \wedge q) \rightarrow (p \vee q)
\]

Show that \( (p \wedge q) \rightarrow (p \vee q) \) is a tautology.
A die is thrown 120 times and getting 1 or 5 is considered a success. Find the mean and variance of the number of successes.

\[(x^3 - 3x^2 + 6x - 6) e^{x} + \log y = c \quad \text{and} \quad yx^3 dx + e^{-x} dy = 0\]

Show that the solution of the differential equation \(yx^3 dx + e^{-x} dy = 0\) is

\[\left( x^3 - 3x^2 + 6x - 6 \right) e^{x} + \log y = c. \]

**PART - IV / IV**

7x5=35

Note: Answer all the questions.

\[x + y + 3z = 0; 4x + 3y + \mu z = 0; 2x + y + 2z = 0\]

(a) For what values of \(\mu\) the system of homogeneous equations \(x + y + 3z = 0; 4x + 3y + \mu z = 0; 2x + y + 2z = 0\) have:

(i) only trivial solution

(ii) infinitely many solutions

OR

(b) Prove by vector method that

\[\sin (A + B) = \sin A \cos B + \cos A \sin B\]
(a) Find the cartesian equation of the plane containing the line $rac{x-2}{2} = \frac{y-2}{3} = \frac{z-1}{-2}$ and passing through the point $(-1, 1, -1)$.

OR

(b) Solve: $x^{11} - x^6 + x^5 - 1 = 0$.

(a) Show that the sum of the focal distances of any point on an ellipse is equal to the length of the major axis and also prove that the locus of a point which moves so that the sum of its distances from $(3, 0)$ and $(-3, 0)$ is 9, is

$$\frac{x^2}{81} + \frac{y^2}{45} = 1.$$

OR

(b) Prove that the area of the largest rectangle that can be inscribed in a circle of radius 'r' is $2r^2$. 
(a) A missile fired from ground level rises \( x \) metres vertically upwards in \( t \) seconds and \( x = 100t - \frac{25}{2}t^2 \). Find:

(i) the initial velocity of the missile

(ii) the time when the height of the missile is a maximum

(iii) the maximum height reached

(iv) the velocity with which the missile strikes the ground

OR

(b) Find the centre, foci and vertices of the hyperbola \( 16x^2 - 9y^2 - 32x - 18y + 151 = 0 \) and draw the diagram.
(a) The mean score of 1000 students for an examination is 34 and the standard deviation is 16. Determine the limit of the marks of the central 70% of the candidates by assuming the distribution is normal.

\[ P[0 < Z < 1.04] = 0.35 \]

(b) Compute the area between the curve \( y = \sin x \) and \( y = \cos x \) and the lines \( x = 0 \) and \( x = \pi \).

(a) If \( w = x + 2y + z^2 \) and \( x = \cos t; \ y = \sin t; \ z = t \) find \( \frac{dw}{dt} \) by using chain rule. Also find \( \frac{dw}{dt} \) by substitution of \( x, y \) and \( z \) in \( w \) and hence verify the result.

(b) A cup of tea at temperature 100°C is placed in a room whose temperature is 15°C and it cools to 60°C in 5 minutes. Find its temperature after further interval of 5 minutes.
(a) State all the five properties of groups.

OR

(b) Prove that the solution of the differential equation:

\[(5D^2 - 8D - 4)y = 5e^{-\frac{2}{5}x} + 2e^x + 3\]

is

\[y = Ae^{2x} + Be^{\frac{-2}{5}x} - \frac{5}{12}xe^{\frac{-2}{5}x} - \frac{2}{7}e^x - \frac{3}{4}\]

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