PART - III

dravidadoss / MATHEMATICS

(മലയാളം, ഇംഗ്ലീഷ് ഉപയോഗം / Malayalam & English Versions)

Instructions:
(1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

Note:
(i) Answer all the questions.
(ii) Choose the most suitable answer from the given four alternatives and write the option code and corresponding answer.
1. If $A$ is a matrix of order 3, then $\det(kA)$ is:
   (1) $k^3 \det(A)$  (2) $k^2 \det(A)$  (3) $k \det(A)$  (4) $\det(A)$

2. $A = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}$
   (1) $\begin{bmatrix} 0 & 0 \\ 0 & 60 \end{bmatrix}$  (2) $\begin{bmatrix} 0 & 0 \\ 0 & 5^{12} \end{bmatrix}$  (3) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  (4) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

3. Given $ae^x + be^y = c; pe^x + qe^y = d$ and
   $\Delta_1 = \begin{vmatrix} a & b \\ p & q \end{vmatrix}; \Delta_2 = \begin{vmatrix} c & b \\ d & q \end{vmatrix}; \Delta_3 = \begin{vmatrix} a & c \\ p & d \end{vmatrix}$
   Find the value of $(x, y)$.
   (1) $\begin{pmatrix} 2 \Delta_2 & \Delta_3 \\ \Delta_1 & \Delta_1 \end{pmatrix}$  (2) $\begin{pmatrix} \log \frac{\Delta_2}{\Delta_1}, \log \frac{\Delta_3}{\Delta_1} \end{pmatrix}$
   (3) $\begin{pmatrix} \log \frac{\Delta_1}{\Delta_3}, \log \frac{\Delta_1}{\Delta_2} \end{pmatrix}$  (4) $\begin{pmatrix} \log \frac{\Delta_1}{\Delta_2}, \log \frac{\Delta_1}{\Delta_3} \end{pmatrix}$
4. In echelon form, which of the following is incorrect?

(1) Every row of A which has all its entries 0 occurs below every row which has a non-zero entry.

(2) The first non-zero entry in each non-zero row is 1.

(3) The number of zeros before the first non-zero element in a row is less than the number of such zeros in the next row.

(4) Two rows can have same number of zeros before the first non-zero entry.

5. If \( \mathbf{PR} = 2 \mathbf{i} + \mathbf{j} + \mathbf{k} \), \( \mathbf{QS} = -\mathbf{i} + 3\mathbf{j} + 2\mathbf{k} \) then the area of the quadrilateral PQRS is:

(1) \( 5\sqrt{3} \) \hspace{1cm} (2) \( 10\sqrt{3} \) \hspace{1cm} (3) \( \frac{5\sqrt{3}}{2} \) \hspace{1cm} (4) \( \frac{3}{2} \)
6. If a line makes $45^\circ, 60^\circ$ with positive direction of axes $x$ and $y$ then the angle it makes with the $z$-axis is:

(1) $30^\circ$  
(2) $90^\circ$  
(3) $45^\circ$  
(4) $60^\circ$

7. The value of $\left[ \vec{i} + \vec{j}, \vec{j} + \vec{k}, \vec{k} + \vec{i} \right]$ is equal to:

(1) $0$  
(2) $1$  
(3) $2$  
(4) $4$

8. The equation of the line parallel to $3x - 3y + 5z = 1$ and passing through the point $(1, 3, 5)$ in vector form, is:

(1) $\vec{r} = \left( \vec{i} + 5\vec{j} + 3\vec{k} \right) + t\left( \vec{i} + 3\vec{j} + 5\vec{k} \right)$

(2) $\vec{r} = \left( \vec{i} + 3\vec{j} + 5\vec{k} \right) + t\left( \vec{i} + 5\vec{j} + 3\vec{k} \right)$

(3) $\vec{r} = \left( \vec{i} + 5\vec{j} + 3\frac{\vec{k}}{2} \right) + t\left( \vec{i} + 3\vec{j} + 5\vec{k} \right)$

(4) $\vec{r} = \left( \vec{i} + 3\vec{j} + 5\vec{k} \right) + t\left( \vec{i} + 5\vec{j} + 3\frac{\vec{k}}{2} \right)$
9. \[
\frac{x - 6}{-6} = \frac{y + 4}{4} = \frac{z - 4}{-8}, \quad \frac{x + 1}{2} = \frac{y + 2}{4} = \frac{z + 3}{-2}
\]
The point of intersection of the lines \( \frac{x - 6}{-6} = \frac{y + 4}{4} = \frac{z - 4}{-8} \) and \( \frac{x + 1}{2} = \frac{y + 2}{4} = \frac{z + 3}{-2} \) is:

(1) \((0, 0, -4)\) \quad (2) \((1, 0, 0)\) \quad (3) \((0, 2, 0)\) \quad (4) \((1, 2, 0)\)

10. The non-parametric vector equation of a plane passing through a point whose position vector is \( \vec{a} \) and parallel to \( \vec{u} \) and \( \vec{v} \) is:

(1) \( [\vec{r} - \vec{a}, \vec{u}, \vec{v}] = 0 \) \quad (2) \( [\vec{r}, \vec{u}, \vec{v}] = 0 \)

(3) \( [\vec{r} - \vec{a}, \vec{u} \times \vec{v}] = 0 \) \quad (4) \( [\vec{r}, \vec{u}, \vec{v}] = 0 \)

If \((m - 5) + i(n + 4)\) is the complex conjugate of \((2m + 3) + i(3n - 2)\) then \((n, m)\) are:

(1) \(\left(\frac{-1}{2}, -8\right)\) \quad (2) \(\left(-\frac{1}{2}, 8\right)\) \quad (3) \(\left(\frac{1}{2}, -8\right)\) \quad (4) \(\left(\frac{1}{2}, 8\right)\)
12. P represents \( z \) and if \(|2z - 1| = 2|z|\)
then the locus of \( P \) is:

(1) the straight line \( x = \frac{1}{4} \)  
(2) the straight line \( y = \frac{1}{4} \)  
(3) the straight line \( z = \frac{1}{2} \)  
(4) the circle \( x^2 + y^2 - 4x - 1 = 0 \)

13. \( \omega \) is the cube root of unity then the value of \((1 - \omega)(1 - \omega^2)(1 - \omega^3)(1 - \omega^4)\) is:

(1) 9  
(2) -9  
(3) 16  
(4) 32

14. \( \arg (z) \) -closed interval. Which of the following intervals \( \arg (z) \) lies in?

(1) \([0, \pi/2]\)  
(2) \((-\pi, \pi]\)  
(3) \([0, \pi]\)  
(4) \((-\pi, 0]\)

The principal value of \( \arg (z) \) lies in the interval:

(1) \([0, \pi/2]\)  
(2) \((-\pi, \pi]\)  
(3) \([0, \pi]\)  
(4) \((-\pi, 0]\)
15. \[9x^2 + 5y^2 - 54x - 40y + 116 = 0\] The eccentricity of the conic \[9x^2 + 5y^2 - 54x - 40y + 116 = 0\] is:

\[
\begin{align*}
(1) & & \frac{1}{3} & & (2) & & \frac{2}{3} & & (3) & & \frac{4}{9} & & (4) & & \frac{2}{\sqrt{5}} \\
\end{align*}
\]

16. \[36y^2 - 25x^2 + 900 = 0\] The asymptotes of the hyperbola \[36y^2 - 25x^2 + 900 = 0\] are:

\[
\begin{align*}
(1) & & y = \pm \frac{6}{5}x & & (2) & & y = \pm \frac{5}{6}x & & (3) & & y = \pm \frac{36}{25}x & & (4) & & y = \pm \frac{25}{36}x \\
\end{align*}
\]

17. \[xy = 18\] One of the foci of the rectangular hyperbola \[xy = 18\] is:

\[
\begin{align*}
(1) & & (6, 6) & & (2) & & (3, 3) & & (3) & & (4, 4) & & (4) & & (5, 5) \\
\end{align*}
\]
18. \( y^2 = 4ax \) പരാബോള (Parabola) വാഴ്ത്തു തിരുള്ള \( t_1', \) എന്നാണ് 'ലംബാതീരി' തിരുള്ള \( t_2', \) എന്നാണ് (Parabola) മുൻകലാംപാലിയിൽ \( t_1 + \frac{2}{t_1} \): 

(1) \(-t_2\)  
(2) \(t_2\)  
(3) \(t_1 + t_2\)  
(4) \(\frac{1}{t_2}\)

The normal at \( t_1' \) on the parabola \( y^2 = 4ax \) meets the parabola at \( t_2' \) then \( t_1 + \frac{2}{t_1} \) is:

(1) \(-t_2\)  
(2) \(t_2\)  
(3) \(t_1 + t_2\)  
(4) \(\frac{1}{t_2}\)

19. \( f(x) = \cos \frac{x}{2} \) എന്നാണ് (Rolle’s) രേഖപ്പെടുത്തിയ അഗാധം \( \pi, 3\pi \) എന്നാണ് സ്ഥാനാർഥം അഗാധം? 

(1) 0  
(2) \(2\pi\)  
(3) \(\frac{\pi}{2}\)  
(4) \(\frac{3\pi}{2}\)

The value of 'c' in Rolle’s Theorem for the function \( f(x) = \cos \frac{x}{2} \) on \( \pi, 3\pi \) is:

(1) 0  
(2) \(2\pi\)  
(3) \(\frac{\pi}{2}\)  
(4) \(\frac{3\pi}{2}\)

20. \([0, 3]\) എന്നാണ് \( f(x) = x^2 - 4x + 5 \) എന്നാണ് \( f(x) = x^2 - 4x + 5 \) എന്നാണ് അഗാധം? 

(1) 2  
(2) 3  
(3) 4  
(4) 5

If \( f(x) = x^2 - 4x + 5 \) on \([0, 3]\) then the absolute maximum value is:

(1) 2  
(2) 3  
(3) 4  
(4) 5
21. If \( y = f(x) \) is the function of a curve \((\text{curve})\) at a point \( x_0 \) in the domain \((\text{domain})\), then \( x_0 \) is the point of inflection if the following conditions are met:

\[
\begin{align*}
(1) & \quad f(x_0) = 0 \\
(2) & \quad f'(x_0) = 0 \\
(3) & \quad f''(x_0) = 0 \\
(4) & \quad f'''(x_0) \neq 0
\end{align*}
\]

If \( x_0 \) is the \( x \)-coordinate of the point of inflection of a curve \( y = f(x) \) then (assume second derivative exists):

\[
\begin{align*}
(1) & \quad f(x_0) = 0 \\
(2) & \quad f'(x_0) = 0 \\
(3) & \quad f''(x_0) = 0 \\
(4) & \quad f'''(x_0) \neq 0
\end{align*}
\]

22. The distance-time relationship of a moving body is given by \( y = F(t) \). Then the acceleration (acceleration) \( \dot{y} \) is:

\[
\begin{align*}
(1) & \quad \text{Gradient of the velocity/time graph} \\
(2) & \quad \text{Gradient of the distance/time graph} \\
(3) & \quad \text{Gradient of the acceleration/time graph} \\
(4) & \quad \text{Gradient of the velocity/distance graph}
\end{align*}
\]

23. The curve \( y^2(x-2) = x^2(1+x) \) has:

\[
\begin{align*}
(1) & \quad x \text{ asymptotically approaches } -\infty\text{ as } x \to -\infty \\
(2) & \quad y \text{ asymptotically approaches } -\infty\text{ as } x \to -\infty \\
(3) & \quad 2 \text{ asymptotically approaches } \infty\text{ as } x \to \infty \\
(4) & \quad \text{ no asymptote}
\end{align*}
\]

The curve \( y^2(x-2) = x^2(1+x) \) has:

\[
\begin{align*}
(1) & \quad \text{an asymptote parallel to } x\text{-axis} \\
(2) & \quad \text{an asymptote parallel to } y\text{-axis} \\
(3) & \quad \text{asymptotes parallel to both axes} \\
(4) & \quad \text{no asymptote}
\end{align*}
\]
24. If \( u = f(x, y) \) is a differentiable function of \( x \) and \( y \); where \( x \) and \( y \) are differentiable functions of \( 't' \) then:

\[
\frac{du}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}
\]

If \( u = f(x, y) \) is a differentiable function of \( x \) and \( y \); where \( x \) and \( y \) are differentiable functions of \( 't' \) then:

\[
\frac{du}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}
\]

25. \[
\int_{0}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} \, dx
\]

The value of \( \int_{0}^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1 + \sin x \cos x} \, dx \) is:

(1) \( \frac{\pi}{2} \)  \hspace{1cm} (2) 0  \hspace{1cm} (3) \( \frac{\pi}{4} \)  \hspace{1cm} (4) \( \pi \)
26. \[ \int_{0}^{\frac{\pi}{4}} \cos^3 2x \, dx \] is equal to which of the following?

(1) \( \frac{2}{3} \)  
(2) \( \frac{1}{3} \)  
(3) 0  
(4) \( \frac{2\pi}{3} \)

The value of \( \int_{0}^{\frac{\pi}{4}} \cos^3 2x \, dx \) is:

(1) \( \frac{2}{3} \)  
(2) \( \frac{1}{3} \)  
(3) 0  
(4) \( \frac{2\pi}{3} \)

27. \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is the equation of an ellipse (ellipse) whose eccentricity is given by \( e \). The value of \( e \) depends on the values of \( a \) and \( b \). Which of the following statements is true?

(1) \( b^2 : a^2 \)  
(2) \( a^2 : b^2 \)  
(3) \( a : b \)  
(4) \( b : a \)

Volume of the solid obtained by revolving the area of the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) about major and minor axes are in the ratio:

(1) \( b^2 : a^2 \)  
(2) \( a^2 : b^2 \)  
(3) \( a : b \)  
(4) \( b : a \)

28. \( I_n = \int \cos^n x \, dx \) is evaluated as follows:

(1) \( -\frac{1}{n} \cos^{n-1} x \sin x + \left( \frac{n-1}{n} \right) I_{n-2} \)  
(2) \( \cos^{n-1} x \sin x + \left( \frac{n-1}{n} \right) I_{n-2} \)

(3) \( \frac{1}{n} \cos^{n-1} x \sin x - \left( \frac{n-1}{n} \right) I_{n-2} \)  
(4) \( \frac{1}{n} \cos^{n-1} x \sin x + \left( \frac{n-1}{n} \right) I_{n-2} \)

If \( I_n = \int \cos^n x \, dx \) then \( I_n = \)

(1) \( -\frac{1}{n} \cos^{n-1} x \sin x + \left( \frac{n-1}{n} \right) I_{n-2} \)  
(2) \( \cos^{n-1} x \sin x + \left( \frac{n-1}{n} \right) I_{n-2} \)

(3) \( \frac{1}{n} \cos^{n-1} x \sin x - \left( \frac{n-1}{n} \right) I_{n-2} \)  
(4) \( \frac{1}{n} \cos^{n-1} x \sin x + \left( \frac{n-1}{n} \right) I_{n-2} \)
29. The differential equation \( \left( \frac{dx}{dy} \right)^2 + 5y^3 = x \) is:

(1) of order 2 and degree 1
(2) of order 1 and degree 2
(3) of order 1 and degree 6
(4) of order 1 and degree 3

30. \( y = k e^{\lambda x} \) is:

(1) \( \frac{dy}{dx} = \lambda y \)
(2) \( \frac{dy}{dx} = ky \)
(3) \( \frac{dy}{dx} + ky = 0 \)
(4) \( \frac{dy}{dx} = e^{\lambda x} \)

If \( y = k e^{\lambda x} \) then its differential equation is (where \( k \) is arbitrary constant):

(1) \( \frac{dy}{dx} = \lambda y \)
(2) \( \frac{dy}{dx} = ky \)
(3) \( \frac{dy}{dx} + ky = 0 \)
(4) \( \frac{dy}{dx} = e^{\lambda x} \)

31. \( m < 0 \) is:

(1) \( x = ce^{my} \)
(2) \( x = ce^{-my} \)
(3) \( x = my + c \)
(4) \( x = c \)

Solution of \( \frac{dx}{dy} + mx = 0 \), where \( m < 0 \) is:

(1) \( x = ce^{my} \)
(2) \( x = ce^{-my} \)
(3) \( x = my + c \)
(4) \( x = c \)
32. \[
\frac{dy}{dx} - y \tan x = \cos x
\]

The integrating factor of the differential equation \[
\frac{dy}{dx} - y \tan x = \cos x
\] is:

(1) sec \( x \) (2) cos \( x \) (3) \( e^{\tan x} \) (4) cot \( x \)

33. p - "queryString" T thGæmgæ, q - "queryString" F -do æmgæi ò-Gæke

(i) p \lor q (ii) \neg p \lor q (iii) p \lor \neg q (iv) p \land \neg q

(1) (i), (ii), (iii) (2) (i), (ii), (iv) (3) (i), (iii), (iv) (4) (ii), (iii), (iv)

If p’s truth value is T and q’s truth value is F, then which of the following have the truth value T?

(i) p \lor q (ii) \neg p \lor q (iii) p \lor \neg q (iv) p \land \neg q

(1) (i), (ii), (iii) (2) (i), (ii), (iv) (3) (i), (iii), (iv) (4) (ii), (iii), (iv)

34. \( \omega \) k - "queryString" (k < n):

(1) \( \frac{1}{\omega^k} \) (2) \( \omega^{-1} \) (3) \( \omega^{n-k} \) (4) \( \frac{n}{\omega^k} \)

In the multiplicative group of \( n \) th roots of unity, the inverse of \( \omega^k \) is (k < n):

(1) \( \frac{1}{\omega^k} \) (2) \( \omega^{-1} \) (3) \( \omega^{n-k} \) (4) \( \frac{n}{\omega^k} \)

[ ðøøø / Turn over]
35. \((\mathbb{Z}_9 + \phi)\) の数列 [7] の元の順序 (order) は次のとおりです:

(1) 9  (2) 6  (3) 3  (4) 1

The order of [7] in \((\mathbb{Z}_9 + \phi)\) is:

(1) 9  (2) 6  (3) 3  (4) 1

36. \(\mathbb{Z}/\mathbb{Z}[x^2] / \langle x \rangle\) の元は \(5\) の倍数 {\(x \in \mathbb{Z} / x = 5k + 2, \ k \in \mathbb{Z}\)} の元として以下のとおりです:

(1) [0]  (2) [5]  (3) [7]  (4) [2]

In congruence modulo 5, \{\(x \in \mathbb{Z} / x = 5k + 2, \ k \in \mathbb{Z}\)\} represents:

(1) [0]  (2) [5]  (3) [7]  (4) [2]

37. A random variable \(X\) has the following probability distribution:

<table>
<thead>
<tr>
<th>(X)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(X=x))</td>
<td>(\frac{1}{4})</td>
<td>2a</td>
<td>3a</td>
<td>4a</td>
<td>5a</td>
<td>(\frac{1}{4})</td>
</tr>
</tbody>
</table>

Then \(P(1 \leq X \leq 4)\) is:

(1) \(\frac{10}{21}\)  (2) \(\frac{2}{7}\)  (3) \(\frac{1}{14}\)  (4) \(\frac{1}{2}\)

A random variable \(X\) has the following probability distribution:

<table>
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<tr>
<th>(X)</th>
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<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(X=x))</td>
<td>(\frac{1}{4})</td>
<td>2a</td>
<td>3a</td>
<td>4a</td>
<td>5a</td>
<td>(\frac{1}{4})</td>
</tr>
</tbody>
</table>

Then \(P(1 \leq X \leq 4)\) is:

(1) \(\frac{10}{21}\)  (2) \(\frac{2}{7}\)  (3) \(\frac{1}{14}\)  (4) \(\frac{1}{2}\)
38. The mean of a binomial distribution is 5 and its standard deviation is 2. Then the values of \( n \) and \( p \) are:

\[
\begin{align*}
(1) & \quad \left( \frac{4}{5}, 25 \right) \\
(2) & \quad \left( 25, \frac{4}{5} \right) \\
(3) & \quad \left( \frac{1}{5}, 25 \right) \\
(4) & \quad \left( 25, \frac{1}{5} \right)
\end{align*}
\]

39. The random variable \( X \) follows normal distribution \( f(x) = c e^{-\frac{(x-100)^2}{25}} \). Then the value of \( c \) is:

\[
\begin{align*}
(1) & \quad \sqrt{2\pi} \\
(2) & \quad \frac{1}{\sqrt{2\pi}} \\
(3) & \quad 5\sqrt{2\pi} \\
(4) & \quad \frac{1}{5\sqrt{2\pi}}
\end{align*}
\]

40. A continuous random variable \( X \) has p.d.f. \( f(x) \), then:

\[
\begin{align*}
(1) & \quad 0 \leq f(x) \leq 1 \\
(2) & \quad f(x) \geq 0 \\
(3) & \quad f(x) \leq 1 \\
(4) & \quad 0 < f(x) < 1
\end{align*}
\]
Note: (i) Answer any ten questions.
(ii) Question No. 55 is compulsory and choose any nine from the remaining.

41. Find the rank of the matrix

\[
\begin{bmatrix}
0 & 1 & 2 & 1 \\
2 & -3 & 0 & -1 \\
1 & 1 & -1 & 0
\end{bmatrix}
\]

42. Find the inverse of the matrix

\[
\begin{bmatrix}
3 & 1 & -1 \\
2 & -2 & 0 \\
1 & 2 & -1
\end{bmatrix}
\]

43. Find the point of intersection of the line passing through the two points (1, 1, -1) ; (-1, 0, 1) and the \(xy\)-plane.
44. (i) \( \vec{a} \times \vec{b} = \vec{c} \times \vec{d}, \quad \vec{a} \times \vec{c} = \vec{b} \times \vec{d} \) then show \( \vec{a} - \vec{d}, \quad \vec{b} - \vec{c} \) are parallel.

(ii) \((2, -3, 1), (3, 1, -2)\) are points and \( \vec{a} = \vec{OA} \) and \( \vec{b} = \vec{OB} \) are vectors then \( \vec{a} \times \vec{b} \) is parallel to \( \vec{OC} \).

(i) If \( \vec{a} \times \vec{b} = \vec{c} \times \vec{d} \) and \( \vec{a} \times \vec{c} = \vec{b} \times \vec{d} \), show that \( \vec{a} - \vec{d} \) and \( \vec{b} - \vec{c} \) are parallel.

(ii) Find the direction cosines of the line joining \((2, -3, 1)\) and \((3, 1, -2)\).

45. \( \alpha, \beta \) are complex conjugates of each other and \( \alpha = -\sqrt{2} + i \) then find \( \alpha^2 + \beta^2 - \alpha \beta \).

If \( \alpha \) and \( \beta \) are complex conjugates to each other and \( \alpha = -\sqrt{2} + i \) then find \( \alpha^2 + \beta^2 - \alpha \beta \).

46. \( 7 + 9i, -3 + 7i, 3 + 3i \) are points which represent complex numbers and \( \alpha^2 + \beta^2 - \alpha \beta \) is found.

Show that the points representing the complex numbers \( 7 + 9i, -3 + 7i, 3 + 3i \) form a right angled triangle on the Argand diagram.

47. A particle of unit mass moves so that displacement after \( t \) seconds is given by \( x = 3 \cos (2t - 4) \). Find the acceleration and kinetic energy at the end of 2 seconds.

\[ \text{K.E.} = \frac{1}{2} mv^2, \text{m is mass} \]
48. (i) \[ x^5 (4-x) \] -এর ক্রিয়াসমূহ গণনা করুন।

(ii) \( y = e^x \) -এর রাসায়নিক অংশ থেকে অন্তরিক্ষ করুন।

(i) Find the critical numbers of \( x^5 (4-x) \).

(ii) Determine the domain of convexity of \( y = e^x \).

49. একটি বৃত্তাকার চাপার পরিমাণ 0.02 আমে.নি. এর সাব্যস্ত সরঞ্জামের জন্য পরিমাণ হলো ২৪ অক্টোবর। কাজ করুন। পরিমাণ পরিমাণটি আমে.নি. এর একক সমান্তরাল চাপার পরিমাণ হলো একাধিক ছোট পরিমাণ। (relative) পরিমাণ বৃত্তাকার চাপার (differentials) গণনাকর্ম করুন।

The radius of a circular disc is given as 24 cm. with a maximum error in measurement of 0.02 cm. Estimate the maximum error in the calculated area of the disc and compute the relative error by using differentials.

50. \( y = (D^2 - 4D + 1) x^2 \).

Solve : \( (D^2 - 4D + 1) y = x^2 \)

51. \( q \lor [p \lor (\neg q)] \) সত্যাবাদী সংজ্ঞায়িত / সাধারণভাবে সত্যাবাদী করুন।

Verify whether the statement \( q \lor [p \lor (\neg q)] \) is a tautology or a contradiction.

52. \((p \land q) \lor (\neg r)\) সত্যাবাদী সংজ্ঞায়িত / সাধারণভাবে সত্যাবাদী।

Construct the truth table for \((p \land q) \lor (\neg r)\).
53. (i) Let $Z$ be a standard normal variate. Find the value of $c$ if $P (Z < c) = 0.05$. Here $P [0 < Z < 1.65] = 0.45$.

(ii) Let $X$ be a Binomial variate with parameters $n$ and $p$. Find $n$ if $P (X = 3) = 0.25$.

(iii) The difference between the mean and the variance of a Binomial distribution is 1 and the difference between their squares is 11. Find $n$.

54. A die is tossed twice. A success is getting an odd number on a toss. Find the mean and the variance of the probability distribution of the number of successes.

55. (a) Find the equation of the hyperbola if the centre is $(2, 5)$; the distance between the directrices is 15; the distance between the foci is 20 and the transverse axis is parallel to $y$-axis.

(b) Find the volume of the solid obtained by revolving the loop of the curve $2ay^2 = x(x-a)^2$ about $x$-axis. Here $a > 0$. [Evaluate]
Note: (i) Answer any ten questions.
(ii) Question No. 70 is compulsory and choose any nine from the remaining.

56. Solve,
\[ x + y + 2z = 4 \]
\[ 2x + 2y + 4z = 8 \]
\[ 3x + 3y + 6z = 12 \]
by using determinant method.

57. \[ \cos(A + B) = \cos A \cos B - \sin A \sin B \] (vector method) vector method.
\[ \cos(A + B) = \cos A \cos B - \sin A \sin B : \text{prove by vector method.} \]

58. Find the vector and Cartesian equations of the plane passing through the points with position vectors \( 3 \mathbf{i} + 4 \mathbf{j} + 2 \mathbf{k}, 2 \mathbf{i} - 2 \mathbf{j} - \mathbf{k}, \) and \( 7 \mathbf{i} + \mathbf{k}. \)
59. \[ x^4 - x^3 + x^2 - x + 1 = 0 \]

Solve: \[ x^4 - x^3 + x^2 - x + 1 = 0 \]

60. On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 mts when it is 6 mts away from the point of projection. Finally it reaches the ground 12 mts away from the starting point. Find the angle of projection.

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61. The ceiling in a hallway 20 ft wide is in the shape of a semi ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.

The ceiling in a hallway 20 ft wide is in the shape of a semi ellipse and 18 ft high at the centre. Find the height of the ceiling 4 feet from either wall if the height of the side walls is 12 ft.

62. \[ x + 2y - 5 = 0 \] Find the equation of the rectangular hyperbola which has for one of its asymptotes the line \[ x + 2y - 5 = 0 \] and passes through the points \((6, 0)\) and \((-3, 0)\).

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63. \[ y^2 = 2x \] Find the point on the parabola \(y^2 = 2x\) that is closest to the point \((1, 4)\).

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64. If \( u = \frac{x}{y^2} - \frac{y}{x^2} \) then verify that \( \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} \).

If \( u = \frac{x}{y^2} - \frac{y}{x^2} \), then verify that \( \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} \).

65. \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) is an ellipse. Find the area of the region bounded by the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), by integration.

Find the area of the region bounded by the ellipse \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \), by integration.

66. \( x = a(t - \sin t), \ y = a(1 - \cos t) \) for \( 0 \leq t \leq \pi \).

Find the length of the curve \( x = a(t - \sin t), \ y = a(1 - \cos t) \) between \( t = 0 \) and \( t = \pi \).

Find the length of the curve \( x = a(t - \sin t), \ y = a(1 - \cos t) \) between \( t = 0 \) and \( t = \pi \).

67. A cup of coffee at temperature \( 100^\circ C \) is placed in a room whose temperature is \( 15^\circ C \) and it cools to \( 60^\circ C \) in \( 5 \) minutes. Find its temperature after a further interval of \( 5 \) minutes.

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68. \( \omega^3 = 1, \ \omega \neq 1 \) form a group with respect to matrix multiplication.

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix},
\begin{bmatrix}
\omega & 0 \\
0 & \omega
\end{bmatrix},
\begin{bmatrix}
\omega^2 & 0 \\
0 & \omega
\end{bmatrix},
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix},
\begin{bmatrix}
0 & \omega^2 \\
\omega & 0
\end{bmatrix},
\begin{bmatrix}
0 & \omega \\
\omega^2 & 0
\end{bmatrix}
\]

Show that \( \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix},
\begin{bmatrix}
\omega & 0 \\
0 & \omega
\end{bmatrix},
\begin{bmatrix}
\omega^2 & 0 \\
0 & \omega
\end{bmatrix},
\begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix},
\begin{bmatrix}
0 & \omega^2 \\
\omega & 0
\end{bmatrix},
\begin{bmatrix}
0 & \omega \\
\omega^2 & 0
\end{bmatrix} \), where \( \omega^3 = 1, \ \omega \neq 1 \) form a group with respect to matrix multiplication.
69. \[ f(x) = \begin{cases} 30x^4 e^{-6x^5} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases} \]

\[ f(x) \] is a p.d.f. Therefore, p.d.f. of \( f(x) \) is p.d.f. 

\[ F(1) \] is?

Verify \[ f(x) = \begin{cases} 30x^4 e^{-6x^5} & ; x > 0 \\ 0 & ; \text{otherwise} \end{cases} \] for p.d.f. If \( f(x) \) is a p.d.f. then find \( F(1) \).

70. (a) \[ 2x + 3y = 6 \] to find the equation of the circle \( x^2 + y^2 = 52 \) to find the circle \( x^2 + y^2 = 52 \) and \( x^2 + y^2 = 52 \) to find the circle \( x^2 + y^2 = 52 \) which are parallel to the straight line \( 2x + 3y = 6 \).

(b) Solve the differential equation \( (x+y)^2 \frac{dy}{dx} = a^2 \).

\( -000- \)